Math 656 • Midterm Exam • March 8, 2016 • Victor Matveev

1) (15pts) Find all values of z in polar or Cartesian form, and plot them as points in the complex plane:

(a) $\left(1-i\sqrt{3}\right)^{3/4}$ (b) $\cos z=-2$

2) (15pts) Sketch the image of the region $\{z\in\mathbb{C} : 1\le|z|\le e,\ \operatorname{Im} z\ge 0\}$ under the mapping $w=i\operatorname{Log}(iz)$. You may consider this transform as a sequence of 3 separate, simple steps. Hint: use polar form for the original variable *z*, and note the slight complication from the fact that Log(z) is the branch with $\arg z\in(-\pi/2,\pi/2]$

3) (25pts) Calculate each integral over the indicated circle, or explain why the integral equals zero:

a) $\oint\limits_{|z|=3}\frac{dz}{\left(e^z+1\right)^9}$ b) $\oint\limits_{|z|=5}\frac{e^z\,dz}{\left(e^z+1\right)^9}$ c) $\oint\limits_{|z|=2}\frac{\sin\left(z^2\right)dz}{z^2-2iz-1}$ d) $\oint\limits_{|z|=R}\frac{dz}{\sqrt{z}}$ e) $\int\limits_{|z|=R}\bar{z}\,dz$

4) (15pts) Find the bound on $\left|\int_C\frac{\cosh z}{z^2+2iz-1}dz\right|$, where the integration contour *C* is a straight line connecting points *z*=3*i* and *z*=3. Hint: express cosh *z* in terms of exponentials.

============================ Pick 2 problems between 5, 6, 7 ===========================

5) (15pts) Consider *any* branch of function $f(z)=\left(\frac{z}{z-1}\right)^{1/2}$, describe its branch cut(s) and describe the jump discontinuity of this function across the branch cut(s). Finally, use this branch to compute $f(i)$

6) (15pts) Can the function $f(z)=e^{i\theta(z)}$ be analytic on domain *D* if $\theta(z)$ is a real non-constant function in *D*? Use any method or theorem you like to answer this question.

7) (15 pts) Solve the boundary value problem for the Laplace's equation $\nabla^2\Phi=0$ in an infinite strip, with boundary conditions indicated below (Φ is a real function). Hint: consider analytic functions of form $f(z)=Ae^{kz}$, where *A* and *k* are real constants. Make sure to satisfy all four boundary conditions!